This is an advanced first course in commutative algebra. It will be geared toward the potential AG applications, but it can also be a standalone course for those interested in CA in its own right.

Background needed: Roughly the material of 122/123, but mostly rings, modules, and fields.

Material covered: My goal is to get through chapters 1-13 of Eisenbud. The topics include:

- Localization
- Primary decomposition
- Hilbert's Nullstellensatz
- Artin-Rees Lemma
- Flat families and Tor
- completions of rings
- Noether Normalization
- Systems of parameters
- DVRs
- Dimension Theory
- Hilbert-Samuel Polynomials

Conventions / Motivation

All our rings (unless otherwise specified) will be commutative w/ multiplicative identity 1.

A <u>ring homomorphism</u> is a morphism of abelian groups that preserves multiplication and takes the identity to the identity.

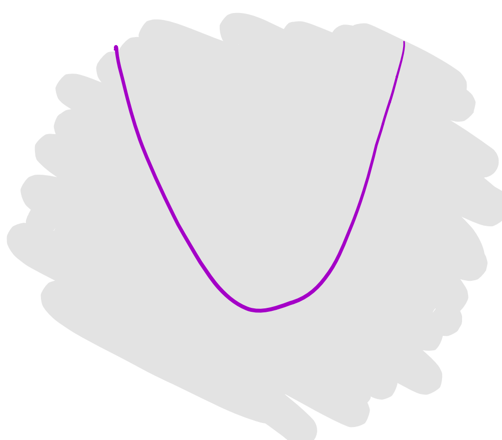
R a ring, $I \neq R$ an ideal is <u>prime</u> if $fg \in I \Rightarrow f$ or $g \in I$. I is <u>maximal</u> if it's not contained in any other proper ideals.

R is a <u>local ring</u> if it has exactly one maximal ideal.

Commutative algebra is intimately connected to algebraic geometry because the algebraic properties of a ring are reflected in geometric properties of the corresponding geometric object (e.g. variety, scheme) and vice versa.

A very geometric collection of examples in CA are polynomial rings over a field $k = \bar{k}$. i.e. $R = k[x_1, \dots, x_n]$.

In this case, the zero set of a polynomial, say $x_1^2 - x_2$ is a locus in k^n ($= \mathbb{A}^n$, "affine n-space")



And in fact, the prime ideals in $k[x_1, \dots, x_n]$ correspond to "subvarieties" of affine space, called "affine varieties".

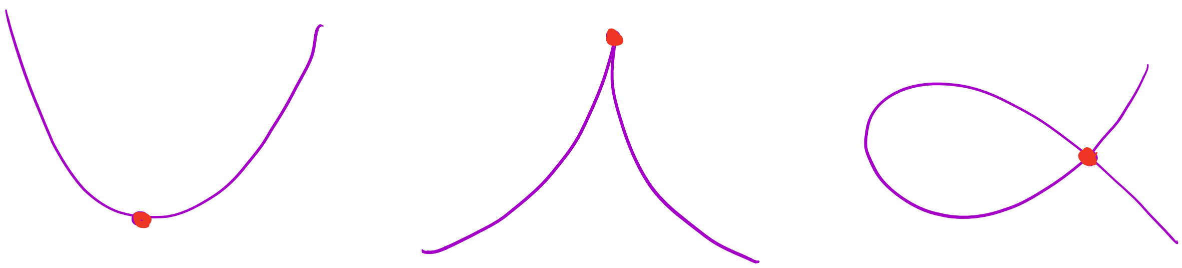
More generally, the prime ideals of an arbitrary ring

R correspond to the points of the "scheme" corresponding to R, called an "affine scheme".

All varieties and schemes can be constructed by gluing together affine varieties/schemes, so, in a way, commutative algebra can be thought of as "local algebraic geometry".

Local rings also have a geometric interpretation: they can describe the geometry of a scheme near a point.

e.g.



a smooth point, cusp, and node all appear to be different geometrically, and, in fact, this is reflected in their corresponding "local rings".

We will make this all more precise soon.